Harmonic scale transformation in effective QFTs with underlying structures

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Abstract

New equations governing the scale transformation behaviors of a QFT with underlying structures are derived. These equations, with their several equivalent versions, can yield some new and significant insights and results that are difficult to see in the conventional renormalization programs. Among the several equivalent versions, one is similar to the usual Callan-Symanzik equation and renormalization group equation but with different meanings. From another version, the anomalous equation of energy-momentum tensor trace can be easily obtained. It can be shown that with the new versions one could partially fix the scheme dependence and hence some renormalization schemes like momentum space subtractions are questionable. The asymptotic freedom for QCD is easily reproduced at one-loop level in our new version CSE. Finally the decoupling theorem a la Appelquist and Carrazone are discussed within our strategy.

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1 Introduction

According to the standard point of view, the QFTs beset with UV unphysical infinities could be seen as corresponding to the effective sectors of the well-defined and complete underlying theory without UV (as well as IR) troubles. The present QFTs are just ill-defined formulations of the well-defined sectors in the underlying theory. From this point of view we proposed a natural strategy for calculating radiative corrections without introducing ad hoc regularizations and UV infinities [1, 2].

In this report, we will derive the equations governing the scale transformation behavior of the QFTs with the existence of underlying structures for making the UV (to focus on the main topic we assume from now on the IR ends are IR regular) ends of the effective QFTs well-defined. Our derivation does not require anything about the renormalizability as will be clear below. Thus the equations obtained are valid for all QFTs as long as they are consistent and/or physically relevant. For the renormalizable cases, the new equations are just new versions of the conventional Callan-Symanzik equations [3].

This paper is organized in the following way: In section two we derive the equations governing the scale behavior of an arbitrary QFT with the understanding that we work in an underlying theory version of it where all objects are UV (and IR) well defined. Then the Callan-Symanzik (CS) like equation is given together with new equivalent versions in section three. The improvements are very obvious and nontrivial. The anomalous trace equation of the energy-momentum tensor follows naturally from a new version of the equations as its operatoral form. In section four, we will elaborate on the harmony revealed by the new scale WT identities and its implications for the conventional renormalization schemes. The conventional result of asymptotic freedom for QCD is also reproduced at 1-loop level. The scheme dependence of the decoupling theorem à la Appelquist and Carrazone [4] is discussed in section five in the underlying theory point of view. All these new achievements clearly demonstrate the power and rationality of our proposal. In section six we summarize our presentation.

2 Underlying structures and scale anomalies

Before starting the derivation, we briefly review our strategy and its consequences: Suppose that the complete and well-defined underlying theory is found, then it must contain certain additional parameters or constants characterizing the correct UV structures underlying the present QFTs. The latter can be now formulated without any (UV) ill-definedness in the underlying theory. As the underlying structures are too small comparing to the scales of the effective theories, the latter are effectively defined in the limit that the underlying structures vanish. But this does not mean that the underlying structures are totally decoupled. On the contrary, as pointed out in Ref. [1, 2], the underlying structures do "influence" quite nontrivially through certain agent constants that are not included in the present QFT formulations. The well-defined formulation of the QFTs are in fact defined in terms of by the canonical model constants of QFTs AND the agent constants. There is no room in our proposal for divergence and counter terms. Hence the meaning of all the finite constants in our proposal is different from the conventionally 'renormalized' ones.

Now let us begin our derivation. Consider a general complete vertex function (or a 1PI Green function), $\Gamma^{(n)}([p],[g];\{\sigma\})$ given in the underlying theory with $[p],[g]$ denoting the external momenta and the Lagrangian couplings (including masses) and $\{\sigma\}$ denoting the underlying parameters or constants. Now it is easy to see that any well-defined vertex function **must** be a homogeneous function of all its dimensionful arguments, that is

$$\Gamma^{(n)}([\lambda p],[\lambda^{d_g} g];\{\lambda^{d_\sigma}\sigma\}) = \lambda^{d_{\Gamma^{(n)}}}\Gamma^{(n)}([p],[g];\{\sigma\}) \tag{1}$$

where $d_{\dots}$ refers to the canonical scale dimension of a parameter or canonical mass dimension of a constant (effective and/or underlying).

In the limit ($L_{\{\sigma\}}$) that the underlying structures vanish, there will necessarily arise some well-defined agent constants $\{\bar{c}\}$ [1, 2], then Eq.(1) becomes

$$\begin{aligned} & \Gamma^{(n)}([\lambda p],[\lambda^{d_g} g];\{\lambda^{d_{\bar{c}}}\bar{c}\}) \equiv L_{\{\sigma\}}\Gamma^{(n)}([\lambda p],[\lambda^{d_g} g];\{\lambda^{d_\sigma}\sigma\}) \\ = \ & \lambda^{d_{\Gamma^{(n)}}}\Gamma^{(n)}([p],[g];\{\bar{c}\}) \equiv \lambda^{d_{\Gamma^{(n)}}} L_{\{\sigma\}}\Gamma^{(n)}([p],[g];\{\sigma\}). \end{aligned} \tag{2}$$

Note that these constants only appear in the loop diagrams of certain vertex functions or in the pure quantum corrections. The limit operation will lead to a local part of certain vertex functions with their coefficients as functions of the agent constants $\bar{c}$ and the Largrangian constants [1]. In all the following deductions, it is understood that for any equation derived in the underlying version ($\Gamma(\cdot;\{\sigma\})$) there is a paralell version defined in terms of the agent constants $\{\bar{c}\}$. Let us work in the underlying version first.

Now the differential equation version of Eq. (1) reads

$$\{\sum_{i=1}^{n} p_i \cdot \partial_{p_i} + \sum d_g g \partial_g + \sum d_\sigma \sigma \partial_\sigma - d_{\Gamma^{(n)}}\} \Gamma^{(n)}([p],[g];\{\sigma\}) = 0. \quad (3)$$

All the scale or mass dimensions are canonical here. Obviously the constants with zero canonical mass dimensions do not cause any violation of the scale bahavior. Eqs. (1) to (3) are valid for any QFT provided they are mathematically consistent and/or physically effective. Renormalizability is simply not in concern here. It is clear that the scale anomalies are caused by the indispensible existence of the underlying structures.

Note that the operation $d_g g \partial_g$ in the above equations inserts certain local operators into the vertex function under consideration, which is in turn equal to the trace of the energy-momentum tensor (Θ) for the QFTs in concern (it is an easy excercise to check this). Thus we arrive at the following,

$$\begin{aligned} & \{\sum_{i=1}^{n} p_i \cdot \partial_{p_i} + \sum d_\sigma \sigma \partial_\sigma - d_{\Gamma^{(n)}}\} \Gamma^{(n)}([p],[g];\{\sigma\}) \\ = \; & i\Gamma^{(n)}_{\Theta}([0;p],[g];\{\sigma\}) \end{aligned} \quad (4)$$

Eq.(3) or equivalently Eq.(4) is just the most general underlying structure version of the Ward-Takahashi identity for scale transformation in (effective) QFTs. The anomalies in the effective QFTs are **canonical behaviors** in the underlying theory point of view as the underlying constants $\{\sigma\}$ are featuring constants there. The paralell equation of Eq. (3) in terms of the agent constants $\{\bar{c}\}$ that can be derived from Eq.(2) read

$$\{\sum_{i=1}^{n} p_i \cdot \partial_{p_i} + \sum d_{\bar{c}} \bar{c} \partial_{\bar{c}} + \sum d_g g \partial_g - d_{\Gamma^{(n)}}\} \Gamma^{(n)}([p],[g];\{\bar{c}\}) = 0. \quad (5)$$

Eqs. (3), (4) and (5) are valid not only order by order but also graph by graph. It is very clear that the agent constants, which arise obviously from

[1]Here the notation $\bar{c}$ takes a different meaning in constrast to Ref. [1, 2] where we use $\bar{c}$ to denote just the definite coefficients of the local part of a vertex function. Later we will denote the coefficients of the local vertices as $\{c\}$.

the limit operation $L_{\{\sigma\}}$, take over all the responsibilities originally held by the underlying structures $\{\sigma\}$. **They are inherent in the well-defined forms of the QFTs. They reside not in the tree graph section of the theories but only in the pure quantum loop graphs**.

Now we note the following obvious fact: Since the agent constants are arguments of the coefficients ($c_{O;\gamma}([g];\{\bar{c}\})$) of the local part of certain vertex functions' loop graph (denoted as γ) due to the limit operation, the variations in these agent constants finally lead to the insertions of the local vertex operators ($[O]$). Some of these composite vertex operators are just given by the Lagrangian. Then the insertion of some of these operators can be realized through the variation with respect to the couplings in the Lagrangian (the insertion of the kinetic terms and others that can not be found in the Lagrangians will be generally denoted as $\hat{I}_O$). That is, with the following facts valid for the complete vertex functions,

$$\delta_{O;\gamma}([g];\{\bar{c}\}) \equiv D_{c_{O;\gamma}}([g];\{\bar{c}\}) \equiv \sum_{\{\bar{c}\}} d_{\bar{c}} \frac{\bar{c}\partial c_{O;\gamma}}{c_{O;\gamma}\partial \bar{c}}, \tag{6}$$

$$\sum d_{\bar{c}}\bar{c}\partial_{\bar{c}} = \sum_{\{O;\gamma\}} D_{c_{O;\gamma}} c_{O;\gamma}\partial_{c_{O;\gamma}} = \sum_{\{c_L\}} D_{c_L} c_L \partial_{c_L} + \sum_{\{c_N\}} D_{c_N} c_N \partial_{c_N}, \tag{7}$$

$$\sum_{\{c_L\}} D_{c_L} c_L \partial_{c_L} = \sum_{[g]} \delta_g([g];\{\bar{c}\}) g\partial_g + \sum_{\{\phi\}} \delta_\phi([g];\{\bar{c}\})\hat{I}_\phi, \tag{8}$$

$$\sum_{\{c_N\}} D_{c_N} c_N \partial_{c_N} = \sum_{\{O_N\}} \delta_{O_N}([g];\{\bar{c}\})\hat{I}_{O_N}, \tag{9}$$

we can turn Eq.(5) into the following forms,

$$\{\lambda\partial_\lambda + \sum_{\{c_{O;\gamma}\}} D_{c_{O;\gamma}} c_{O;\gamma}\partial_{c_{O;\gamma}} + \sum_{[g]} d_g g\partial_g - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p],[g]|\{c_{O;\gamma}\}) = 0, \tag{10}$$

$$\begin{aligned} &\{\lambda\partial_\lambda + \sum_{\{O_N\}} \delta_{O_N}\hat{I}_{O_N} + \sum_{[g]}(\delta_g + d_g)g\partial_g + \sum_{\{\phi\}} \delta_\phi \hat{I}_\phi \\ &-d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p],[g]|\{c_{O;\gamma}\}) = 0, \end{aligned} \tag{11}$$

Here the insertion of the kinetic vertex of a field ϕ is denoted as $\hat{I}_\phi$. $\delta_{\dots}$ or $D_{c_{\dots}}$ denote the coefficients before the inserted composite operators. (A vertex function's dependence upon the agent constants is effected through the local coefficient constants after the vertical bar.) Eq.(10) is true order by order, graph by graph while Eq.(11) is only true for the complete sum of all graphs (or sum up to a certain order). The constants $\{c_{...}\}$ must satisfy all the above equations as functions of the agent constants **and** the model constants that are classically defined. Thus one can view the above equations as constraints imposed by the most natural fact–homogeneity–upon these constants. Moreover, the above equations are readily true for the connected but one-particle-reducible correlation functions of fields (and even composite operators) as long as one works in the underlying theory version or its effective theory limit.

Although in Eq.(11) we have rewritten some of anomalies in terms of the insertions of the composite operators appearing in the Lagrangian, they are specifying the scale behaviors of the theories's **quantum structures**, not that of the classical constants. In fact they describe the influences of the "normal" scale behaviors of the underlying structures unpon the effective QFTs. Later, in section four, we will discuss more closely about this point followed by some significant conequences which have not been achieved in the conventional renormalization programs.

3 New versions of Callan-Symanzik equation and RGE

In this section we limit our attention to a special type of theories where there is no variations in certain local vertices (those denoted by $\sum D_{c_N} c_N \partial_{c_N}$ or $\sum_{\{O_N\}} \delta_{O_N}([g];\{\bar{c}\}) \hat{I}_{O_N}$ in Eq.(9)) that can not be realized through insertions of the Lagrangian operators. In conventional terminology, we consider in this section only renormalizable theories.

Without the variations necessarily described by $\sum D_{\bar{c}_N} \bar{c}_N$, Eqs. (10) and (11) should take the following form,

$$\{\lambda\partial_\lambda + \sum d_g g \partial_g + \sum_{\{c_L\}} D_{c_L} c_L \partial_{c_L} - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p],[g]|\{c_{...}\}) = 0, \tag{12}$$

$$\{\lambda\partial_\lambda + \sum (\delta_g + d_g) g \partial_g + \sum \delta_\phi \hat{I}_\phi - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p],[g]|\{c_{...}\}) = 0, \tag{13}$$

These are just some new versions of the CS equations for renormalizable theories. Before turning them into more familar forms,let us first make the following remarks.

As pointed out in section two, the most striking difference between $[g]$ and $\{\bar{c}\}$ is that the former are defined in the **tree level vertices** while the latter only appear in **loop amplitudes**. The classical constants $[g]$ or the tree level vertices alone can not yield a well-defined definition of the quantum structures of fields for theories with potential UV divergence[2]. But the incompleteness of the constants $[g]$ or the unavailability of $\{\sigma\}$ by now does not mean that there must be divergence. Of course among $\{\sigma\}$ or their agents $\{\bar{c}\}$ that characterize the quantum "paths", there must be ones with mass dimensions specifying the effective quantum motions' scale(s). In our opinion, this is the physical basis of the necessary appearance of a mass or energy scale in any regularization/renormalization scheme. In our strategy, these scales naturally appear in the course of indefinite integrations.

On the other hand, the locality of the appearance of these quantum agents does allow a mix among $[g]$ and the agent constants (through $\{c_\gamma([g];\{\bar{c}\})\}$) **for the complete** vertex functions or their generating functional in renormalizable theories. Suppose there are scale transforms in the agent constants, then

$$\begin{aligned} &\sum_{\{\bar{c}\}} d_{\bar{c}}\bar{c}\partial_{\bar{c}}\Gamma^{(n)}(\cdots|\{c_{...}([g];\{\bar{c}\})\}) = \sum_{\{c_{...}\}} D_{c_{...}}c_{...}\partial_{c_{...}}\Gamma^{(n)}(\cdots|\{c_{...}([g];\{\bar{c}\})\}) \\ = &\ \{\sum_{[g]}\delta_g g\partial_g + \sum_{\{\phi\}}\delta_\phi \hat{I}_\phi\}\Gamma^{(n)}(\cdots|\{c_{...}([g];\{\bar{c}\})\}) \end{aligned} \tag{14}$$

which is equivalent to

$$\begin{aligned} &\{\sum_{\{\bar{c}\}} d_{\bar{c}}\bar{c}\partial_{\bar{c}} - \sum_{\{c_{...}\}} D_{c_{...}}c_{...}\partial_{c_{...}}\}\Gamma^{(n)}(\cdots|\{c_{...}([g];\{\bar{c}\})\}) \\ = &\ \{\sum_{\{\bar{c}\}} d_{\bar{c}}\bar{c}\partial_{\bar{c}} - \sum_{[g]}\delta_g g\partial_g - \sum_{\{\phi\}}\delta_\phi \hat{I}_\phi\}\Gamma^{(n)}(\cdots|\{c_{...}([g];\{\bar{c}\})\}) = 0. \end{aligned} \tag{15}$$

It is easy to see that the "invariance" in Eq.(15) is in fact a rearrangement of the classically defined parts and that arising from quantum loops or a

[2]In this sense, the present standard quantization procedures are not absolutely well-defined as they have not been able to specify the quantum agent constants for such theories. For more detailed elaboration of this point please refer to Ref.[5].

tautology of the fact that agent constants' changes only affect the elementary like local vertices as evident in Eq.(14). This is our new version of the renormalization group equation.

Now let us turn the kinetic terms' contribtuion into other forms. This can be achieved by noting that the coefficient constants in the kinetic vertices only "renormalize" or rescale the line momenta in the graphs. For example, for the fermions, this is just the rescaling of $i\not{p} \rightarrow (1+c_\psi)i\not{p}$ [3]. Similarly for bosons, $ip^2 \rightarrow (1+c_\phi)ip^2$. Since the lines must end up in various vertices, such rescaling of lines would effectively lead to the rescaling of vertices and be shared by the latter in pairs. Thus a vertex is typically rescaled as $\prod_i(1+c_{\psi_i})^{-1/2}\prod_j(1+c_{\phi_j})^{-1/2}$ depending on the number of line momenta that join it. For an n-point 1PI vertex function there must be n external momenta flow into or out of a number of elementary vertices in the graphs. As the external momenta are not subject to the rescaling there must be compensated rescaling for these vertices containing external momenta–an overall rescaling of the complete n-point vertex functions. Thus we have

$$\begin{array}{lcl} \delta_g & \rightarrow & \bar{\bar{\delta}}_g \equiv (\delta_g - n_{g;\phi}\dfrac{\delta_\phi}{2} - n_{g;\psi}\dfrac{\delta_\psi}{2}), \quad d_g = 0; \\ \delta_g & \rightarrow & \bar{\delta}_g \equiv (\delta_g - \delta_{\psi or \phi}), \quad d_g \neq 0; \\ \Gamma^{(n_\phi, n_\psi)} & \rightarrow & (1+c_\psi)^{n_\psi/2}(1+c_\phi)^{n_\phi/2}\Gamma^{(n_\phi, n_\psi)}. \end{array} \tag{16}$$

Then the above equations take the following forms:

$$\begin{array}{l} \{\lambda\partial_\lambda + \sum(\bar{\delta}_g + d_g)g\partial_g + n_\phi\dfrac{\delta_\phi}{2} + n_\psi\dfrac{\delta_\psi}{2} - d_{\Gamma^{(n_\phi,n_\psi)}}\} \\ \times\Gamma^{(n_\phi,n_\psi)}([\lambda p],[g]|\{c_{\dots}\}) = 0, \end{array} \tag{17}$$

$$\{\sum_{\{\bar{c}\}} d_{\bar{c}}\bar{c}\partial_{\bar{c}} - \sum_{[g]}\bar{\delta}_g g\partial_g - n_\phi\frac{\delta_\phi}{2} - n_\psi\frac{\delta_\psi}{2}\}\Gamma^{(n_\phi,n_\psi)}([p],[g]|\{c_{\dots}\}) = 0. \tag{18}$$

One can see that Eqs.(17) and (18) take exactly the same form as the usual CSE and RGE. But we must emphasize again here that everything here is

[3] We express such finite renormalization in terms of the various vertices and lines appearing in graphs as these are the original places where the agent constants or the coefficient constants really appear.

given in terms of the classical constants $[g]$ and agent constants. One might take the $[g]$ as our finite "bare" constants.

Now some remarks are in order:

(1). Each equation is given in terms of the classically defined constants $[g]$ and the quantum agents definitely defined in the complete underlying theory, no divergence and hence no infinite renromalization is needed. In the conventional schemes, divergence is inevitable due to the artificiality of the regularization methods and hence infinite renormalization is necessary. Then after subtraction, one needs to **"reidentify"** the originally classically defined constants with a product of two singular objects–bare quantities and the renormalization constants.

(2). Conventionally, the infinite bare quantities are in fact regularizarion scheme dependent–different divergnce in different schemes, therefore scheme dependence is inherent there as infinite renormalization only transforms the expressions not the essence. In the underlying theory point of view this problem is due to the artificiality of the regularization schemes for simulating the true underlying structures [4]. There should be only one physical scheme, the one with the true agent constants selected by the nature together with the classically defined Lagrangian constants apart from some redefinition freedom for the special type theories described in Eqs. (14), (15) or (18).

Let us take QED as an example for demonstration.

The CS equation and RGE in conventional renormalization schemes read respectively [6]

$$\begin{aligned}&\{\lambda\partial_\lambda + (1+\gamma_{m_R})m_R\partial_{m_R} - \beta\partial_{\alpha_R} + n_A\gamma_A - n_\psi\gamma_\psi + d_{\Gamma^{(n_A,n_\psi)}}\}\\ &\times\Gamma^{(n_A,n_\psi)}([\lambda p], m_R, \alpha_R) = 0; \end{aligned} \quad (19)$$

$$\{\mu\partial_\mu - \gamma_{m_R}m_R\partial_{m_R} + \beta\partial_{\alpha_R} - n_A\gamma_A - n_\psi\gamma_\psi\}\Gamma^{(n_A,n_\psi)}([p], m_R, \alpha_R) = 0. \quad (20)$$

While to write down our new version CSE and RGE for QED we need to be specific on the following aspects: Among the agent constants there is at

[4]Conventioanlly, it is believed that the scheme dependence is only inherent in the perturbative truncations but not in the full renormalized theory, basing on the expectation that the low energy theories are independent of the short distance structures and hence independent of regularization schemes. This is not correct as even the underlying constants do not show up in the effective theory, they do affect the effective theories through their agent constants. The scale anomalies are just the evidences of the influences of the underlying structures. Otherwise, why bother choosing among regularizations?

least one mass scale parameter, we will denote it as $\bar{\mu}$ and parametrize all the agents into one mass scale and a series of constants $\{\bar{c}_{i;0}\}$ with zero mass dimention so that the coefficients take the following form

$$\begin{aligned} c_m &= C_m(m, e; \bar{\mu}) + C_{m;0}(\{\bar{c}_{i;0}\}), \\ c_e &= C_e(m, e; \bar{\mu}) + C_{e;0}(\{\bar{c}_{i;0}\}) = c_\psi, \\ c_A &= C_A(m, e; \bar{\mu}) + C_{A;0}(\{\bar{c}_{i;0}\}). \end{aligned} \tag{21}$$

where the Ward-Takahashi identity for gauge invariance is used. Then from Eqs.(12) to (18) we can write down our new CSE and RGE in several equivalent versions as follows

$$\begin{aligned} & \{\lambda\partial_\lambda + m\partial_m + D_{c_m} c_m \partial_{c_m} + D_{c_A} c_A \partial_{c_A} + D_{c_\psi} c_\psi \partial_{c_\psi} - d_{\Gamma^{(n_A, n_\psi)}}\} \\ & \times\Gamma^{(n_A, n_\psi)}([\lambda p], m, e|\{c_m, c_e, c_A, c_\psi\}) \\ = & \{\lambda\partial_\lambda + (1+\delta_m) m\partial_m + \delta_e e\partial_e + \delta_A \hat{I}_A + \delta_\psi \hat{I}_\psi - d_{\Gamma^{(n_A, n_\psi)}}\} \\ & \times\Gamma^{(n_A, n_\psi)}([\lambda p], m, e|\{c_m, c_e, c_A, c_\psi\}) \\ = & \{\lambda\partial_\lambda + (\bar{\delta}_m + 1) m\partial_m + \frac{\bar{\delta}_e}{2} e\partial_e + n_A \frac{\delta_A}{2} + n_\psi \frac{\delta_\psi}{2} - d_{\Gamma^{(n_A, n_\psi)}}\} \\ & \times\Gamma^{(n)}([\lambda p], m, e|\{c_m, c_e, c_A, c_\psi\}) = 0; \end{aligned} \tag{22}$$

$$\begin{aligned} & \{\bar{\mu}\partial_{\bar{\mu}} - D_{c_m} c_m \partial_{c_m} - D_{c_A} c_A \partial_{c_A} - D_{c_\psi} c_\psi \partial_{c_\psi}\} \\ & \times\Gamma^{(n_A, n_\psi)}([p], m, e|\{c_m, c_e, c_A, c_\psi\}) \\ = & \{\bar{\mu}\partial_{\bar{\mu}} - \delta_m m\partial_m - \delta_e e\partial_e - \delta_A \hat{I}_A - \delta_\psi \hat{I}_\psi\} \\ & \times\Gamma^{(n_A, n_\psi)}([p], m, e|\{c_m, c_e, c_A, c_\psi\}) \\ = & \{\bar{\mu}\partial_{\bar{\mu}} - \bar{\delta}_m m\partial_m - \frac{\bar{\delta}_e}{2} e\partial_e - n_\phi \frac{\delta_\phi}{2} - n_\psi \frac{\delta_\psi}{2}\} \\ & \times\Gamma^{(n_A, n_\psi)}([p], m, e|\{c_m, c_e, c_A, c_\psi\}) = 0. \end{aligned} \tag{23}$$

Now we can compare our new CSE and RGE in (22) and (23) with the conventional ones in (19) and (20). The anomalies are all described in our equations in terms of the classical constants m, e or (α) and the agents $\bar{\mu}, \bar{c}...$, while in the conventional renormalization schemes the anomalies are given in terms of the renormalized constants m_R, α_R depending on an arbitrary scale μ via the infinite renormalization constants. To the lowest order the correspondence should be

$$\gamma_{m_R} \sim \bar{\delta}_m \equiv (\delta_m - \delta_\psi); \quad \beta(\alpha_R) = 2\gamma_A \sim \delta_A \sim -2\bar{\delta}_e; \quad 2\gamma_\psi \sim \delta_\psi. \tag{24}$$

We note again that the equations expressed in terms of the coefficient constants directly are valid order by order and graph by graph as well as for the complete sum or sum up to a certain order while the ones given in terms of vertex insertions are only valid for the complete sum or sum up to certain order (so it is with the conventional CSE and RGE).

Now we have derived the equations governing the scale transform behavior of QFTs with underlying structures. The new equations are totally given in terms of classically defined model constants and the agent constants that have not been predicted but are extremely important for a complete definition of the "effective" QFTs. No divergence should be there at all which are artefacts of ad hoc regularizations. We have given several equivalent forms of equations, one only in terms of the agent constants, one in terms of local vertex insertions and one takes the usual form of the conventional CSE and RGE but with no infinite renormalization. Each is suitable for different purposes. The utility and power of the conventional form of CSE and RGE has been well known. Below and in section four we will demonstrate the utility of the other two versions that has not expected in the conventional frameworks.

From the experiences above it is also easy to derive the WT identity of scale transformation for generating functional of vertex functions of a QFT from homogeneity,

$$
\begin{aligned}
&\{\sum_{\{\phi\}}\int d^Dx[d_\phi - x\cdot\partial_x)\phi(x)]\frac{\delta}{\delta\phi(x)} + \sum_{[g]} d_g g\partial_g + \sum_{\{\bar{c}\}} d_{\bar{c}}\bar{c}\partial_{\bar{c}} - D\} \\
&\times\Gamma^{1PI}([\phi],[g];\{\bar{c}\}) \\
= \quad &\{\sum_{\{\phi\}}\int d^Dx[d_\phi - x\cdot\partial_x)\phi(x)]\frac{\delta}{\delta\phi(x)} + \sum_{[g]}(d_g+\delta_g)\partial_g + \sum_{\{\phi\}}\delta_\phi\hat{I}_\phi - D\} \\
&\times\Gamma^{1PI}([\phi],[g];\{\bar{c}\}) = 0. \qquad (25)
\end{aligned}
$$

with D denoting the spacetime dimension. Again this is correct for any consistent QFTs.

To exhibit the utility of the version in terms of the operator insertion, we simply note that once it is translated into the operatoral form we get the anomalous trace equation of quantum energy-momentum tensor since all the quantum effects all well defined here and all the effects have been given in terms of known quantum operators. Thus one can just read the trace equation from the operator insertion version of Eq.(25), for QED this is simply

$$\begin{aligned} g_{\mu\nu}\Theta^{\mu\nu} &= (1+\delta_m)m\bar{\psi}\psi + \frac{1}{4}\delta_A F^{\mu\nu}F_{\mu\nu} - \delta_\psi i\bar{\psi}\gamma_\mu D^\mu\psi \\ &= (1+\delta_m-\delta_\psi)m\bar{\psi}\psi + \frac{1}{4}\delta_A F^{\mu\nu}F_{\mu\nu}. \end{aligned} \tag{26}$$

In the last step we have used the motion equation. Considering the correpondence in Eq. (24), this is exactly the operatoral trace anomaly equation for QED [7] in a new version without divergence and subractions. In unrenormalizable theories, the trace anomalies would contain an infinite sum of local composite operators beyond the Lagrangian operators.

In next section we will illustrate the correctness of our new WT identity or new CS equation in unconventional versions in several examples and demonstrate their physical utility and significance.

4 Harmony contraint on the agent constants

From section two we see that the well-defined vertex functions or their generating functional for a QFT must be homogeneous with respect to **all** the dimensional parameters and constants which must include the underlying structures or their agents. That is to say, for this **homogeneity** to be valid in a well-defined formulation the underlying structures are indispensible. Therefore in the underlying theory point of view the scale transform is in perfect harmony in the following sense: When all the arguments scale up together according to their normal scale or mass dimensions the vertex functions(al) must then exhibit exact scale behaviors. This is the harmony principle we wish to advocate that is understandable only in terms of underlying structures.

This harmony leads to a natural explanation for the scale or trace anomaly. On the one hand, in the underlying theory formulation, there is no scale anomaly. The scale behavior is definite and normal there. In the course of scale transformation of all fields and all constants, there should be generally tranformations into each other (just like any symmetry transformation) in a definite way to preserve the overall homogeneity. Then as we only effectively observe the relatively low sectors' variables and their behavior, the nonexact scale variations of the effective variables that should be absorbed by the underlying structures' variations are therefore exposed. Thus it is equally correct to interpret the anomalies as the normal contributions of underlying strucutres and as the anomalous behaviors of the effective sectors that should be unambiguous in terms of the canonical variables of effective theories [5]. They are the two faces of one thing. Chiral anomaly can be understood in similar way [9]. We hope this harmony principle could be generalized to other tranformation behaviors of the effective theories.

All our derivations and arguments has been presented with the assumption that the underlying theory version of the QFTs are known. Of course this is not true. Then we must try to determine them otherwise. However, there are quite many constraints for these unknown constants or to reduce the ambiguities associated with them, say, the novel symmetries of the effective QFT systems. It is known in gauge field theories that gaug invariance reduces the number of ambiguities (or the number of independent divergences in conventional terminology). Here, with the harmony notion elaborated above, further important constraints upon the ambiguities can be imposed. This scale harmony has in fact locked the scale behaviors of the agent constants with that of the definite parts of the effective theories and hence the ambiguities are further reduced. Now let us enumerate some examples to illustrate the validity of our new CS equations and the role of ambiguity reduction of these equations in the notion of harmony.

First let us see that the scale anomalies are definite at 1-loop level (Let us still work in QED) in electron self-energy $\Sigma^{(1)}$, electron-photon vertex $\Lambda^{(1)}_{\mu}$ and photon polarization $\Pi^{\mu\nu}$. For simplicity let us consider the cases of massless electrons. As for the expressions of these objects in terms of the unknown agent constants, one can use any regularization to compute the definite part and just leave the ambiguous local part as an ambiguous polynomial of external momentum. Or one can use the technique introduced in Ref. [1, 2]. (We will also explain this technique later.) It is sufficient to parametrize the agant constants with one dimensional constant $\bar{\mu}$ and a

[5] We had shown elsewhere [8] and will further demonstrate below that scale or trace anomaly is in fact due to the presence of a kind of rational terms that are nonlocal (hence definite at least in 1-loop level). The presence of this kind of rational terms is unambiguous or independent of regularization schemes. It is in this sense the anomaly is a definite property of QFT.

number of dimensionless constants. Then from homogeneity we find that

$$\Sigma^{(1)}(p,-p) = -i\frac{e^2}{16\pi^2}\not{p}[ln\frac{p^2}{\bar{\mu}^2} + c^0_\psi(\{\bar{c}^0\})];$$
$$\Lambda^{(1)}_\nu(p,0|p) = +i\frac{e^2}{16\pi^2}\gamma_\nu[ln\frac{p^2}{\bar{\mu}^2} + 2\not{p}p_\nu/p^2 + c^0_e(\{\bar{c}^0\})];$$
$$\Pi^{(1)\mu\nu}(p,-p) = i\frac{e^2}{12\pi^2}(p^2g^{\mu\nu} - p^\mu p^\nu)[ln\frac{p^2}{\bar{\mu}^2} + c^0_A(\{\bar{c}^0\})]. \quad (27)$$

Here all the constants are meant for 1-loop case and the superscript 0 of the constants means that these constants are dimensionless and independent of any dimansional constants and we work in Feynman gauge. Then these objects have scale anomalies in terms of canonical variables only (here only momentum)

$$\{p\cdot\partial_p - 1\}\Sigma^{(1)} = -i\frac{e^2}{8\pi^2}\not{p};$$
$$\{p\cdot\partial_p\}\Lambda^{(1)}_\nu = +i\frac{e^2}{8\pi^2}\gamma_\nu;$$
$$\{p\cdot\partial_p - 2\}\Pi^{(1)\mu\nu} = i\frac{e^2}{6\pi^2}(p^2g^{\mu\nu} - p^\mu p^\nu). \quad (28)$$

Now if we include the agent constants' contributions, which is $\bar{\mu}\partial_{\bar{\mu}}$ (the dimensionless agent simply do not affect scale behavior), then we can recover the exact scale behavior or the exact scale harmony in the version with explicit agent constants which is valid order by order, graph by graph:

$$\{p\cdot\partial_p + \bar{\mu}\partial_{\bar{\mu}} - 1\}\Sigma^{(1)} = 0;$$
$$\{p\cdot\partial_p + \bar{\mu}\partial_{\bar{\mu}}\}\Lambda^{(1)}_\nu = 0;$$
$$\{p\cdot\partial_p + \bar{\mu}\partial_{\bar{\mu}} - 2\}\Pi^{(1)\mu\nu} = 0. \quad (29)$$

This is just harmony we have explained above. The dimensional agent constant $\bar{\mu}$ must appear in the vertices as $ln\bar{\mu}^2$. They must compensate the unambiguous anomalies from the definite and nonlocal parts. (But the other dimensionless agent constants remain undetermined by the Scale WT identities.) It is easy to see that the anomaly in each 1-loop vertex above is due to the logarithmic dependence upon momentum which upon the action of $p\cdot\partial_p$ yields a local term from a nonlocal one: $(p\cdot\partial_p)lnp^2 = p_\mu(\frac{2p^\mu}{p^2}) = 2$. Or if one starts with the insertion of the dilatation current into the elementary vertices there should be an unbiguous rational term in terms of momentum only like $\sim \frac{p^\mu p_{\dots}}{p^2}$ which yields a local terms after contracting with divergence operation $p_\mu \times \frac{p^\mu \prod p_{\dots}}{p^2} = \prod p_{\dots}$. By the way, we note that the conventional version of CSE and the one in terms of vertex insertions can not hold order by order and graph by graph but only for the complete vertex functions or sums up to certain order.

If one sums the verices up to 1-loop level with tree graph vertices included she/he could get the vertex insertion version as follows,

$$\begin{aligned}
&\{p \cdot \partial_p + \delta_\psi \hat{I}_\psi - 1\}\Gamma^{(1)}_\psi = 0; \\
&\{p \cdot \partial_p + \delta_e e \partial_e\}\Gamma^{(1)}_{e;\mu} = 0; \\
&\{p \cdot \partial_p + \delta_A \hat{I}_A - 2\}\Gamma^{(1)\mu\nu} = 0; \qquad (30) \\
&\delta^{(1)}_\psi = \frac{e^2}{8\pi^2}; \quad \delta^{(1)}_e = \frac{e^2}{8\pi^2}; \quad \delta^{(1)}_A = \frac{e^2}{6\pi^2}. \qquad (31)
\end{aligned}$$

It is easy to see that the anomalies are independent of mass. One can recalculate the 1-loop corrections of the above vertices in the massive case with the technique we proposed [1, 2]. Here we just list the photon polarization case given by Chanowitz and Ellis [10] but parametrize the local parts following our strategy,

$$\begin{aligned}
\Pi^{\mu\nu} &= -i\frac{e^2}{12\pi^2}(p^\mu p^\nu - g^{\mu\nu}p^2)\{c_A(m;\{\bar{\mu}, \bar{c}^0_i\}) \\
&\quad + 6\int_0^1 dz z(1-z) ln[1 - z(1-z)p^2/m^2]\}; \qquad (32) \\
\Delta^{\mu\nu} &= \frac{e^2}{\pi^2}(p^\mu p^\nu - g^{\mu\nu}p^2)\frac{m^2}{p^2}(mF - 1), \\
F &\equiv \frac{2}{(p^4 - 4m^2p^2)^{1/2}} ln\left\{\frac{p^2 - (p^4 - 4m^2p^2)^{1/2}}{p^2 + (p^4 - 4m^2p^2)^{1/2}}\right\}, \qquad (33) \\
\{p \cdot \partial_p - 2\}\Pi^{\mu\nu} &= i\Delta^{\mu\nu} + i\frac{e^2}{6\pi^2}(g^{\mu\nu}p^2 - p^\mu p^\nu) \qquad (34)
\end{aligned}$$

where the anomaly is exactly the same as in the massless case.

Now if we include the $\bar{\mu}\partial_{\bar{\mu}}$ operation amd that of the mass, there should not be any anomaly as all dimensional arguments are harmonically considered. Thus the anomaly must be assumed up by the dimensional agent's variation, or the dimensional agent must be locked in this way with the definite anomaly,

$$\bar{\mu}\partial_{\bar{\mu}} c_A(m; \{\bar{\mu}, \bar{c}_i^0\}) = -2 \tag{35}$$

$$\longrightarrow c_A(m; \{\bar{\mu}, \bar{c}_i^0\}) = -ln\frac{\bar{\mu}^2}{m^2} + c_A(\{\bar{c}_i^0\}) \tag{36}$$

where the only dimensional constant besides the agent $\bar{\mu}$ is fermion mass here [6]. Thus we have locked the behavior of the underlying structures acted by their agent(s). Or each agent constant associated with a ill-defined loop integration must appear in the way expressed in Eq.(36). It is easy to see that all the other elementary vertices should exhibit the same feature. This is quite a natural conclusion following from the standard underlying theory point of view. In a sense the harmony notion elaborated here fixes the agent constants to a certain degree. The fixation of the constants here corresponds to the subtraction procedure in the conventional renormalization programs where the dependence of the renormalized amplitudes upon the subtraction points can be almost arbitrary. Our discussion above simply rejects all the schemes where the dependence of the residual local parts of the elementary vertices upon the subtraction scale is different from that given by Eq.(36) in the first place before referring to other problems of the schemes. This could not have been achieved in the conventional renormalization programs.

Therefore according to the foregoing discussions, the subtraction schemes effected at Euclidean momentum are questionable for the massive case [7]. Here we must mention that the dependence on the agent constants in the way specified in Eq.(36) is conventionally subject to the nondecoupling difficulty [12]. However, this is not a real challenge by noting that the underlying formulation is correct only when the limit $L_{\{\sigma\}}$ is valid with repsect to all effective parameters including the agents. When a mass goes to infinity, the

[6] One can easily see the homogeneous dependence on $\frac{\bar{\mu}^2}{m^2}$ by noting that if the nonpolynomial momentum dependent part has mass to fulfill the homogeneity, then the coefficients must depend on mass in this way. One can also see this point by directly solving the scale WT identity in the agent constants version, i.e., using Eq. (12) or (5) that is valid graph by graph.

[7] This is not to say the other schemes are totally OK. It is known that some regularization schemes like dimensional regularization sometimes leads to oversubtraction and are therefore questionable in certain applications [11]. The most crucial point is that all regularization schemes are ad hoc ones contain various ill-definedness.

expressions that have been obtained by assuming it is finite are no longer true. We will make further dicussions on this issue in section five.

The natural appearance of the dimensional agents in the way dictated in Eq. (36) can also be technically understood in the following way. First we need to briefly review the technique we proposed solely based on the existence of the underlying structures for providing well-defined formulations [1, 2] as follows: For a 1-loop graph G of superficial divergence degree $\omega_G - 1$ [13] with its underlying version amplitude denoted as $\Gamma_G([p],[g];\{\sigma\})$, we have

$$\begin{aligned}
&\partial_{[p]}^{\omega_G} L_{\{\sigma\}} \Gamma_G([p],[g];\{\sigma\}) = L_{\{\sigma\}} \partial_{[p]}^{\omega_G} \Gamma_G([p],[g];\{\sigma\}) \\
= \;& L_{\{\sigma\}} \int d^d k [\partial_{[p]}^{\omega_G} g([p,k],[g];\{\sigma\})] = \int d^d k [\partial_{[p]}^{\omega_G} L_{\{\sigma\}} g([p,k],[g];\{\sigma\})] \\
= \;& \int d^d k g^{\omega_G}([p,k],[g];\{\sigma\}) = \Gamma_G^{\omega_G}([p],[g]), \qquad (37) \\
&\Gamma_G([p],[g];\{c_G([g];\{\bar{c}\})\}) = L_{\{\sigma\}} \Gamma_G([p],[g];\{\sigma\}) \rightarrow \\
&\Gamma_G([p],[g];\{c_G^u\}) \equiv [\partial_{[p]}^{\omega_G}]^{-1} \Gamma_G^{\omega_G}([p],[g]) \qquad (38)
\end{aligned}$$

where $\partial_{[p]}^{\omega_G}$ and its inverse denote the differentiation (for ω times) with respect to the external momenta (one can do the same with respect to masses in certain cases like for tadpole graphs) and the indefinite integration respectively. We can not get the definite agent constants or the coefficient constants but a set of unknown constants $\{c_G^u\}$ instead in our technical treatment. The justification of Eq.(37) is basing on the natural assumption that the underlying structures make the QFTs well defined and the low energy limit does not alter the well-definedness.

Our above analysis implies that these unknown constants are to be defined as the true agents through various physical properties. There must be at least one constant with mass dimension among the constants $\{c_G^u\}$ as the result of the indefinite integration though its precise value are to be fixed otherwise and it must appear in the polynomial part of $\Gamma_G([p],[g];\{c_G^u\})$ as $\sim ln\frac{\mu^2}{m^2}$ where m is a mass among the model constants $[g]$. This can be seen by observing that the amplitude $\Gamma_G^{\omega_G}([p],[g])$ must take the following form (with appropriate Feynman parametrization)

$$\Gamma_G^{\omega_G}([p],[g]) \sim \int_0^1 [dx] f(\{x\}) \frac{l([p],[g_m]|\{x\})}{[Q([p],[g_m]|\{x\})]} \qquad (39)$$

with $l(\cdots)$ and $Q(\cdots)$ denoting respectively the linear and quadratic expressions of the combination of dimensional parameters like external momenta and masses with the help of Feynman parameters $\{x\}$. Then on the first step indefinte integration with respect to the external parameters a logarithmic expression is generated with an arbitrary mass scale entered for balancing the mass dimensions,

$$\int_{indef} dp\Gamma^{\omega_G}_G([p],[g]) \sim \int_0^1 [dx] f(\{x\}) ln \left\{ \frac{Q([p],[g_m]|\{x\})}{\mu^2} \right\}. \quad (40)$$

Further indefinite integrations with respect to the external parameters of mass dimension would necessarily carry on this mass scale in the polynomial part. If one trades the μ in the logarithmic function with a model mass constant so that the nonpolynomial function of momenta only involve model mass parameters, the μ would only appear in the polynomial part in the form of $ln\frac{\mu^2}{m^2}$ just as we claimed through scale anomaly analysis. The multiplying factor of this logarithmic function is nothing else but the "anomalous" dimension:$\delta_{\cdots}([g_0])$. Thus at 1-loop level, for all local ambiguities, we can claim that:

$$c_{\hat{O}_{vert}}([g];\{\bar{\mu},\bar{c}^0_i\}) = \frac{1}{2}\delta_{\hat{O}_{vert}} ln \left\{ \frac{\bar{\mu}^2}{m^2} \right\} + C^0_{\hat{O}_{vert}}(\{\bar{c}^0_i\}). \quad (41)$$

The superscript 0 indicates that its principal is totally independent of any constants with nonzero mass dimension.

Our techincal proposal can be very naturally generalized to the multi-loop cases and many conventional subtlties like overlapping divergences are automatically resolved, please refer to Ref. [2] [8]. We will provide more examples of calculations in the proposed technique in future reports, expecially in the nonperturbative case where the conventional regularization schemes often yield more severe divergences.

Before closing this section we simply demonstrate that the asymptotic freedom of QCD can be easily reproduced in our new version of CSE. The new version CSE for the two-point connected correlation function of gauge

[8] Moreover, according to this technique, the new ambiguities in the potentially divergent multiloop graphs beyond the subgraph ambiguities must appear in the local part as $ln\frac{\mu^2}{m^2}$. Thus summing all the overall local parts associated with each graph would lead to local vertices with coneefficients $\sim [1+\frac{1}{2}\delta_{O_v}([g^0])ln\{\mu^2/m^2\}+C^0_{O_v}(\{\bar{c}^0_i\})]$. That is the underlying structures only "renormalize" the model constants $[g]$ by simple logarithmic dependence on the masses and an agent scale.

fields with external legs amputed reads (a specific gauge like Feynman gauge is assumed)

$$\{\lambda\partial_\lambda + \bar{\delta}_{g_s} g_s \partial_{g_s} + \sum (1+\bar{\delta}_{m_i}) m_i \partial_{m_i} - \delta_A\} D^c(\lambda p, g_s, [m_i]; \{\bar{\mu}, c_j^0\}) = 0. \quad (42)$$

For simplicity let us work in the massless case or the high energy situation. Noting that $\bar{\delta}_{g_s}$ is equal to $-\frac{1}{2}\delta^{(1)}_{A_\mu}$ according to Eq. (16) and rewriting the CSE in terms of $\alpha(= g_s^2/(4\pi))$, we have ($\alpha_{eff} \equiv \alpha D^c$)

$$\begin{aligned} & \{\lambda\partial_\lambda - \delta_A \alpha\partial_\alpha - \delta_A\} D^c(\lambda p, \alpha; \bar{\mu}) = 0 \\ => \quad & \{\lambda\partial_\lambda - \delta_A \alpha\partial_\alpha\} \alpha_{eff}(\lambda p, \alpha; \bar{\mu}) = 0. \end{aligned} \quad (43)$$

Here we have simply assumed that the constants independent of any mass scale is zero or absorbed into the mass scale. Expanding α_{eff} in series of $\frac{1}{2} ln[p^2/\bar{\mu}^2]$ one can find the following relation for one loop result of $\delta_A^{(1)}(= a\alpha)$

$$\alpha^{(1)}_{eff} = \sum_{n=0}^{\infty} f_n(\alpha) t^n, \quad t \equiv \frac{1}{2} ln[p^2/\bar{\mu}^2], \quad (44)$$

$$n f_n = \delta_A^{(1)} \alpha \partial_\alpha f_{n-1}, \quad f_0 = \alpha, \quad \forall n \geq 1. \quad (45)$$

The solution to Eq.(45) is easy to find and hence the solution for the effective coupling depending on the physical energy scale is,

$$f_n = a^n \alpha^{n+1}, \quad \forall n \geq 0, \quad (46)$$

$$\alpha^{(1)}_{eff}(p^2, \alpha; \bar{\mu}) = \frac{\alpha}{1 - \frac{1}{2} a\alpha ln\{\frac{p^2}{\bar{\mu}^2}\}}. \quad (47)$$

The asymptotic freedom is clearly reproduced for QCD due to the negative constant a at one-loop level. The solution of the effective coupling given in Eq.(47) corresponds to the one-particle-REDUCIBLE two-point Green function whose CSE is given in terms of variations in the one-perticle-IRREDUCIBLE vertices. The IR Landau pole in the effective coupling for QCD can not be trusted as we have worked in the UV end with IR details coarsed away. For QED, the Landau pole is UV one (due to positive a), but it is not a final answer before the higher order corrections of δ_A are included. Moreover, as the energy is extremely higher, there is another problem: the underlying structures would "interfere with" the effective sectors more strongly, i.e., the present QFT models must be modified somehow. This is closely related with the validity of the limit operation $L_{\{\sigma\}}$ and will be discussed in more details in next section. Here our solution is trustable at energies not extremely high. It is also worthwhile to note that due to the difference in the definitions of the anomalies, there is **multivaluedness** in the effective couplings solved from the conventional CSE or RGE due to the '**reidentification**' mentioned in section three (remarks following Eq.(18)), while our new CSE is free from this defect.

5 Decoupling theorem and underlying structures

Now let us turn to the decoupling issue of the heavy particles in QFTs.

As we have noted in last section, due to the logarithmic dependence of the local constants (c_{O_i}) upon the particle mass ($\sim lnm^2$,see Eq.(41)) these constants would diverge as the particles become extremely heavy ($m \rightarrow \infty$) and this in turn implies the nondecoupling of the heavy particles. While in the subraction schemes effected at Euclidean momentum, the heavy particles' contributions to the light sector vanish as the masses go to infinity [4, 12].

To resolve this scheme dependence of decoupling phenomenon, we note that from the underlying theory point of view, when a physical mass or mass scale goes to infinity relative to the characteristic energy scale of an effective physical system, it means that the physics associated with this infinite energy scale belongs to the UV underlying sectors of the effective system. Or the heavy particles now join the underlying structures' party and the underlying constants $\{\sigma\}$ are no longer infinities for such heavy particles, then the expressions containing $ln\frac{\mu^2}{m^2}$ that have arisen from the limit operation $L_{\{\sigma\}}$ is no longer valid, or equivalently, the following operation is simply illegal if one really accepts the underlying theory point of view,

$$lim_{m\rightarrow\infty} ln\frac{\mu^2}{m^2}. \tag{48}$$

The correct procedure is now to expand the set of underlying constants $\{\sigma\}$ to include the extremely heavy particles' masses and rewrite the whole QFT in terms of effective parameters of the light particles or fields. Then the limit operation $L_{\{\sigma\}}$ now changes to $L_{\{\sigma;(M_{heavy})\}}$ for the light objects and like other underlying structures, these heavy particles' influences are delegated to the agent constants that finally appear from the new limit operation $L_{\{\sigma;(M_{heavy})\}}$. Since these agants are finite in principle, the effects of the heavy particles only "renormalize" the light sectors by a finite amount in the local vertices. In the sense of (48), it is also easy to see that the expressions obtained at finite momenta for QFTs can not be simply extrapolated to the UV limit as it would again invalidate the limit operation about the underlying structures.

Equivalently, one can let the energy and masses of the light sectors become relatively infinitesimal comparing to the heavy sectors, then the light sectors would be again given in terms of masses and couplings belonging to the light sectors and additional finite agent constants appearing only in the local vertices of the light fields. This is just the spirit behind the heavy quark field theory [12].

As the effetive parameters like mass and momenta become extremely large, such fields would interact more directly with the underlying structures. Then the limit of decoupling the underlying structures for these heavy or high-energy fields are no longer valid and one has to work in models suitable for more higher energy physics. Thus the scheme dependence of the decoupling theorem is again closely connected to the indispensible existence of the underlying structures for making the QFTs naturally well defined. The intimate relation between the decoupling theorem and the existence of the underlying structures will be investigated intensively in the future. Moreover, to make the formulation completely well defined, the IR underlying structures should be included in all the future investigations.

Conventionally, the subtaction point is deemed as arbitrary, a running scale. While in the underlying theory point of view, due to the condition of decoupling theorem or the validity of the LE limit operation, such scales (agent constants) should be uniquely defined up to possible equivalence. One could not arbitrarily rescale any effective parameter (like mass and energy) and any agent constant (which characterizes the quantum structures of an effective sector in addition to the Planck constant and effective parameters, see section II and III). Once a scale is extrapolated to a place where the effective QFT models kreak down, the formulations in terms of the LE effective parameters and agent constants cease to be correct or useful.

6 Summary

From our discussions above, we can see that the underlying theory postulate is quite natural and powerful. Many subtle issues in conventional schemes are resolved in a natural way in our proposal [2]. The new equations governing the scale behaviors are formulated in terms of canonical and agent constants that are well defined. The most significant technical prediction from our investigations here is the irrationality of certain conventional renormalization schemes. We could also reproduce many novel results. The derivation of trace anomaly in the underlying theory approach is a simple work.

As the scheme dependence in high energy applications of QCD is a very important issue, we hope our investigations here could be of help to it. Of course our main goal here has been to demonstrate the plausibility and power of our recent proposal for renormalization, further development and applications of the proposal will be made in the future. We hope our strategy could lead to more important contributions to high energy physics and beyond. The issue of gauge dependence of the CSE in gauge field theories will be discussed in the future together with the IR problem in such QFTs.

In summary, we derived several new versions of the Ward-Takahashi identities of scale transformations in any QFT models along the natural strategy for dealing with the UV ill-definedness. Among these new versions, each has its own advantage and utility in different situations. Our expressions are all free from divergences and the associated infinite renormalization. The new equations improve the conventional ones in several aspects. A most important new result is that the finite local part of any potentially divergent vertex graph can not be arbitrary and therefore some conventional subtraction schemes are, according to the underlying theory point of view, unreasonable choices. The important decoupling theorem a la Applequist and Carrazone is argued to be still valid outside Euclidean momenta subtraction schemes with the underlying structures appropriately accounted.

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